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Renormalization Group Evolution of Dirac Neutrino Masses

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Abstract

There are good reasons why neutrinos could be Majorana particles, but there exist also a number of very good reasons why neutrinos could have Dirac masses. The latter option deserves more attention and we derive therefore analytic expressions describing the renormalization group evolution of mixing angles and of the CP phase for Dirac neutrinos. Radiative corrections to leptonic mixings are in this case enhanced compared to the quark mixings because the hierarchy of neutrino masses is milder and because the mixing angles are larger. The renormalization group effects are compared to the precision of current and future neutrino experiments. We find that, in the MSSM framework, radiative corrections of the mixing angles are for large $\tan\beta$ comparable to the precision of future experiments.

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1 Introduction

One of the most important open questions of neutrino physics is whether neutrinos are Dirac or Majorana particles. From a theoretical perspective, large Majorana mass terms appear quite naturally for the right-handed neutrinos, since they are complete gauge singlets. This leads directly to the best investigated (and therefore most popular) explanations for the huge ratio of observed mass scales in the see-saw mechanism [1–6]. In its simplest form, neutrino masses get suppressed by a factor v_{EW}/M_* with v_{EW} denoting the vacuum expectation value (VEV) of the Higgs boson and M_* being the scale at which $B-L$ symmetry (baryon minus lepton number) is assumed to be broken. However, it is important to keep in mind that the suppression factor v/M_* (with M_* now being the GUT scale or a related scale) arises rather generally whenever neutrino masses arise from integrating out heavy degrees of freedom with mass M_*. This statement holds independently of the nature of neutrino masses, in particular both for Dirac and Majorana masses. Indeed, there exist a couple of appealing models where small Dirac masses are explained in this way by using extra, heavy degrees of freedom, or by relating the Yukawa coupling Y_ν to the ratio of gravitino mass (or other soft masses) and GUT (or compactification) scale [7–12]. Another possibility is using localization in extra dimensions, and explaining the suppression by a small overlap of the corresponding zero-mode profiles along extra dimensions (see, e.g., [13–15]). Further support for Dirac neutrinos was found in certain orbifold compactifications of the heterotic string, where it is difficult to obtain the standard see-saw [16]. For recent overviews and further references of various possibilities of explaining small Dirac masses see [17–19].

Cosmological arguments do not give a preference for Dirac or Majorana masses either. For instance, even if one requires the explanation of the observed baryon asymmetry to be related to neutrino properties, one finds that successful baryogenesis can work both for Majorana [20, 21] and Dirac [22, 23] neutrinos. Dirac neutrinos evade also constraints from primordial nucleosynthesis, since the right-handed degrees of freedom decouple with a low temperature so that their energy density is relatively suppressed [24]. The question whether neutrinos are Dirac or Majorana particles is therefore one of the main motivations for improved neutrino-less double beta decay experiments. Both options should therefore be studied seriously until this question is clarified by experiments.

We investigate in this Letter RG effects under the assumption that neutrinos are Dirac particles, and that the small Yukawa couplings are explained by some mechanism operating at a high, e.g. GUT or compactification, scale, denoted by M_{GUT} in the following. The radiative corrections to the leptonic CP violation has been studied in [25]. We extend this analysis to include all leptonic mixing parameters, and derive analytic formulae describing the renormalization group (RG) running of the leptonic mixing parameters. The radiative corrections are compared to analogous corrections in the quark sector, and we will show that generically RG running in the lepton sector is stronger than in the quark sector since the coefficients of the renormalization group equations (RGEs) are enhanced due to the fact that mass hierarchy is milder and the mixing angles are larger in the lepton sector. We compare the size of the radiative corrections to the accuracy of present and future neutrino experiments, and find that they are particularly relevant if neutrino masses are degenerate and $\tan\beta$ is large in the MSSM.

2 Analytic formulae

Using the standard parameterization (see, e.g., [26]) for leptonic (and quark) mixing the RGEs for the leptonic mixing angles read

$$\dot{\theta}_{12} = \frac{-C\,y_\tau^2}{32\,\pi^2}\,\frac{m_1^2+m_2^2}{m_2^2-m_1^2}\,\sin(2\,\theta_{12})\,\sin^2\theta_{23}+\mathscr{O}(\theta_{13})\;, \tag{1}$$

$$\begin{aligned}\dot{\theta}_{13} = \frac{-C\,y_\tau^2}{32\,\pi^2}\frac{1}{(m_3^2-m_1^2)\,(m_3^2-m_2^2)}\Big\{&\left(m_2^2-m_1^2\right)\,m_3^2\,\cos\delta\,\cos\theta_{13}\,\sin(2\,\theta_{12})\,\sin(2\,\theta_{23})\\ &+\left[m_3^4-\left(m_2^2-m_1^2\right)\,m_3^2\,\cos(2\,\theta_{12})-m_1^2\,m_2^2\right]\,\cos^2\theta_{23}\,\sin(2\,\theta_{13})\Big\}\;,\end{aligned} \tag{2}$$

$$\dot{\theta}_{23} = \frac{-C\,y_\tau^2}{32\,\pi^2}\,\frac{\left[m_3^4-m_1^2\,m_2^2+(m_2^2-m_1^2)\,m_3^2\,\cos(2\,\theta_{12})\right]}{(m_3^2-m_1^2)\,(m_3^2-m_2^2)}\,\sin(2\,\theta_{23})+\mathscr{O}(\theta_{13})\;, \tag{3}$$

where the dot indicates the logarithmic derivative w.r.t. the renormalization scale μ, e.g. $\dot{\theta}_{12} = \mathrm{d}\theta_{12}/\mathrm{d}t = \mu\,\mathrm{d}\theta_{12}/\mathrm{d}\mu$, and

$$C = \begin{cases} 1\;, & \text{(MSSM)}\;,\\ -3/2\;, & \text{(SM)}\;. \end{cases} \tag{4}$$

Here, we have neglected the tiny electron and muon Yukawa couplings, as well as the neutrino Yukawa couplings, against the τ Yukawa coupling y_τ. Furthermore, in $\dot{\theta}_{12}$ and $\dot{\theta}_{23}$ we only kept the leading order term of an expansion in the reactor mixing angle θ_{13}. We have checked numerically that this is a good approximation for realistic values of θ_{13}.

It is instructive to compare Eqs. (1)–(3) to the corresponding expressions for Majorana neutrinos. Technically one obtains Eqs. (1)–(3) by discarding all terms which depend on the Majorana phases in Eqs. (8)–(10) of Ref. [27]. One could thus say that the running of the Dirac mixing parameters resembles the running of Majorana mixing parameters averaged over the Majorana phases φ_1 and φ_2[1]. This means in particular that strong damping effects for the evolution of θ_{12}, as observed for Majorana neutrinos, cannot occur in the Dirac case.

From Eqs. (1)–(3), we can immediately recognize several features of the RG evolution. First, for a strong mass hierarchy, the running of the angles is negligible. For $m_1 = 0$, the angles always run less than $1°$ (except for θ_{23} which runs more if $\tan\beta \gtrsim 40$). Second, for $m_1 \gtrsim 0.02\,\mathrm{eV}$, θ_{12} has the strongest RG evolution. Third, as is obvious from Eqs. (1) and (3), in the MSSM θ_{12} always increases when running downwards while θ_{23} increases for a normal and decreases for an inverted mass hierarchy. This means that these two angles

[1] Stated differently, the running in the Dirac case is roughly half of the maximal running in the Majorana case. The factor 1/2 can be understood from the structure of the RGE: in the Dirac case, the mass matrix gets rotated by only one term (cf. Eq. (A.3d)),

$$\Delta m_\nu = C\,m_\nu\,\left(Y_e^\dagger Y_e\right) + \text{flavor-trivial terms}\;,$$

while in the Majorana case there are two terms

$$\Delta m_\nu = C\,\left[m_\nu\,\left(Y_e^\dagger Y_e\right)+\left(Y_e^\dagger Y_e\right)^T\,m_\nu\right] + \text{flavor-trivial terms}\;,$$

with $C = -3/2$ in SM [28] and two-Higgs models [29], and $C = 1$ in the MSSM [30,31].

are radiatively enhanced (for normal mass ordering) which may, at least partially, be the reason for their large size. Whether θ_{13} increases or decreases depends on δ.

The evolution of the Dirac phase δ is described by[2]

$$\dot{\delta} = \dot{\delta}^{(-1)}\theta_{13}^{-1} + \dot{\delta}^{(0)} + \dot{\delta}^{(1)} + \mathscr{O}\left(\theta_{13}^2\right) , \tag{5}$$

with the first two coefficients $\dot{\delta}^{(k)}$ given by

$$\dot{\delta}^{(-1)} = \frac{C\, y_\tau^2}{32\,\pi^2} \frac{(m_2^2 - m_1^2)\, m_3^2}{(m_3^2 - m_1^2)\,(m_3^2 - m_2^2)} \sin(\delta)\, \sin(2\,\theta_{12})\, \sin(2\,\theta_{23}) , \tag{6a}$$

$$\dot{\delta}^{(0)} = 0 . \tag{6b}$$

Moreover, the term linear in θ_{13} contains

$$\dot{\delta}^{(1)} = \frac{C\, y_\tau^2}{16\,\pi^2} \frac{m_2^2\,(m_3^2 - m_1^2)^2}{(m_2^2 - m_1^2)\,(m_3^2 - m_1^2)\,(m_3^2 - m_2^2)} \cot(\theta_{12})\, \sin(2\,\theta_{23})\, \sin\delta + \dots , \tag{6c}$$

which becomes relevant if θ_{13} is not too small.

As usual, δ is undefined for $\theta_{13} = 0$. Clearly, if δ vanishes for some scale, the (lepton sector of the) theory is CP invariant at this scale and thus remains CP invariant for all scales. Hence, the statement $\delta = 0$ cannot depend on the renormalization scale, which can also be seen in our formulae. Likewise, if θ_{13} is zero at some given scale, the theory must again be CP invariant for all scales[3]. From this we conclude that θ_{13} can never cross zero if we have at some scale $\theta_{13} \neq 0$ and $\sin\delta \neq 0$. If θ_{13} approaches zero, then we can see from (5) that δ runs quickly to a value such that the coefficient in (2) changes its sign and θ_{13} increases again. Thus, the only case where θ_{13} can cross zero is the CP conserving case. This is interesting for future precision measurements of θ_{13}, since the assumption of leptonic CP violation at any scale leads to the conclusion that the weak-scale value of θ_{13} does not vanish. We illustrate the corresponding large effects in the evolution of δ in Fig. 1. For all our plots, we use the software packages `REAP` and `MPT` associated with [33].

We can understand this feature also differently. In the above approximation, we can write $U_{e3} = \theta_{13} e^{-\mathrm{i}\delta}$ and by inserting the RGEs for θ_{13} and δ, we find in the limit $\theta_{13} \to 0$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} U_{e3} &= \dot{\theta}_{13}\, e^{-\mathrm{i}\delta} - \mathrm{i}\,\theta_{13}\, e^{-\mathrm{i}\delta}\, \dot{\delta} \\ &\simeq \frac{C\, y_\tau^2}{32\,\pi^2} \frac{(m_2^2 - m_1^2)\, m_3^2}{(m_3^2 - m_1^2)\,(m_3^2 - m_2^2)} \sin(2\,\theta_{12})\, \sin(2\,\theta_{23}) . \end{aligned} \tag{7}$$

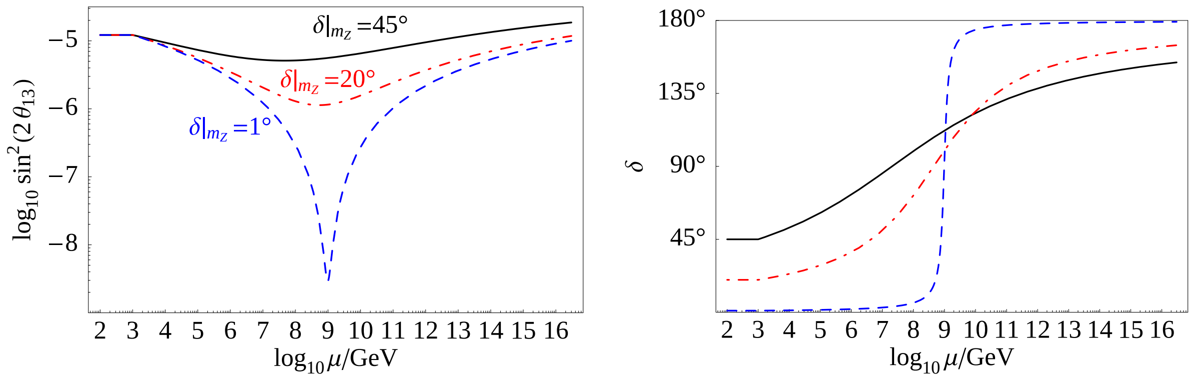
For $\theta_{13} \to 0$ we find thus that the RG change of U_{e3} is along the real axis and U_{e3} can therefore only become zero if it is real. The imaginary part of Eq. (7) allows to determine a minimal value of θ_{13} as $(\theta_{13})_{\min} \simeq \theta_{13}(\mu)\, \sin\delta(\mu)$ where any scale μ can be used.

Furthermore, let t_0 denote the turning point of θ_{13} characterized by $\delta = \pi/2$. The 'asymptotic' behavior $\delta(t - t_0) \simeq -\delta(t_0 - t) = \pi - \delta(t_0 - t)$ is a consequence of the fact that $\dot{\delta}$ is an odd function in θ_{13}[4]. This allows to understand why δ approaches $\pi - \delta(m_Z)$ for large μ in Fig. 1.

[2]The evolution of the weak basis CP invariant has already been studied in [25].

[3]This is in contrast to the case of Majorana neutrinos, where the memory to CP violation can be stored in the Majorana phases, and θ_{13} can cross zero even in the presence of leptonic CP violation [27,32].

[4]We have introduced π in order to keep θ_{13} positive as we use the convention that θ_{13} is always positive, and a possible sign flip is absorbed in a jump of δ.



(a) $\sin^2(2\theta_{13})$. (b) δ.

Figure 1: Evolution of (a) $\sin^2(2\theta_{13})$ and (b) δ for small values of θ_{13}. We use $\tan\beta = 50$, $\theta_{13} = 0.1°$, $m_1 = 0.1\,\mathrm{eV}$ and best-fit values for all other parameters. The solid (black), dash-dotted (red) and dashed (blue) curve shows the evolution of (a) θ_{13} and (b) δ for $\delta = 45°$, $20°$ and $1°$ at the electroweak scale, respectively. θ_{13} cannot become 0 in any of the cases.

The evolution of the mass eigenvalues is given by

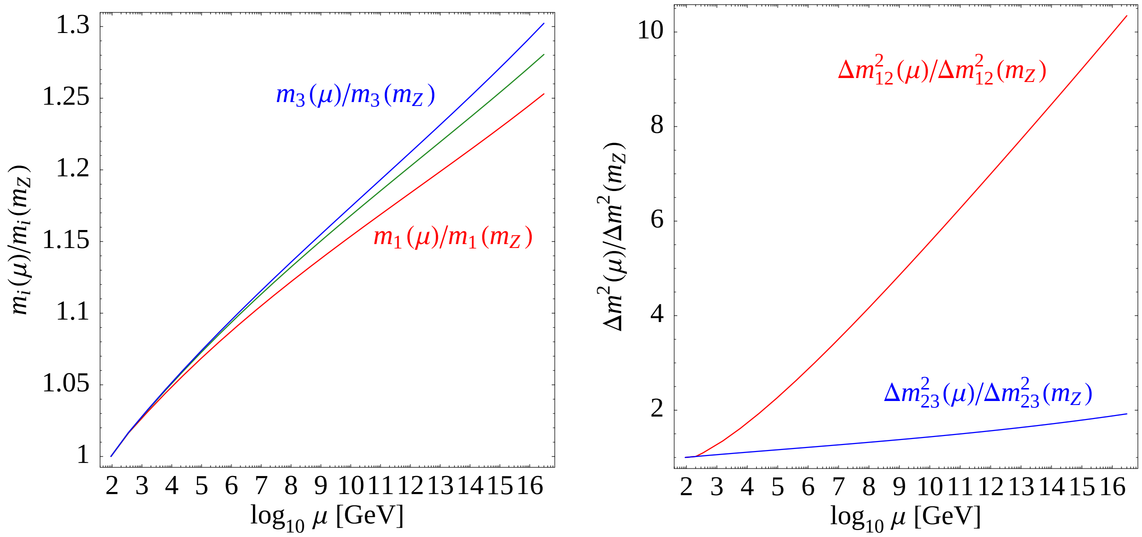
$$16\pi^2\,\dot{m}_1 = \Big\{C\,y_\tau^2\Big[\cos^2\theta_{12}\,\cos^2\theta_{23}\,\sin^2\theta_{13}+\sin^2\theta_{12}\,\sin^2\theta_{23} - \frac{1}{2}\cos\delta\,\sin\theta_{13}\,\sin(2\,\theta_{12})\,\sin(2\,\theta_{23})\Big]+\alpha_\nu\Big\}\,m_1\;, \tag{8a}$$

$$16\pi^2\,\dot{m}_2 = \Big\{C\,y_\tau^2\Big[\sin^2\theta_{12}\,\cos^2\theta_{23}\,\sin^2\theta_{13}+\cos^2\theta_{12}\,\sin^2\theta_{23} + \frac{1}{2}\cos\delta\,\sin\theta_{13}\,\sin(2\,\theta_{12})\,\sin(2\,\theta_{23})\Big]+\alpha_\nu\Big\}\,m_2\;, \tag{8b}$$

$$16\pi^2\,\dot{m}_3 = \big\{C\,y_\tau^2\,\cos^2\theta_{13}\,\cos^2\theta_{23}+\alpha_\nu\big\}\,m_3\;. \tag{8c}$$

α_ν represents the flavor-independent part of the RGE, and is given in (A.5d). Clearly, the dominant RG effect of the masses is a common rescaling governed by α_ν. In addition, for large $\tan\beta$ in the MSSM, there are corrections specific to the individual m_i. In leading order in θ_{13}, these effects tend to decrease $\Delta m^2_{\mathrm{atm}}$ for a normal hierarchy and to increase $\Delta m^2_{\mathrm{atm}}$ for an inverted hierarchy when running down.

Fig. 2 shows an extreme example of the evolution of the mass eigenvalues and the corresponding $\Delta m^2_{\mathrm{sol}}$ and $\Delta m^2_{\mathrm{atm}}$. For large $\tan\beta$, there are substantial deviations from the flavor-independent scaling of the mass eigenvalues. The latter can be approximately inferred from the curve of m_1 in Fig. 2 (a).



(a) $m_i(\mu)/m_i(m_Z)$.

(b) $\Delta m^2_{ij}(\mu)/\Delta m^2_{ij}(m_Z)$.

Figure 2: Example of the evolution of the mass eigenvalues and the Δm^2s. We choose $m_1(m_Z) = 0.1\,\mathrm{eV}$, $\delta(m_Z) = 50°$, $\tan\beta = 50$ and a SUSY breaking scale of $200\,\mathrm{GeV}$, and best-fit values otherwise.

3 Radiative corrections and future precision experiments

An important question is if future experiments will reach a precision which allows to draw interesting conclusions from quantum corrections. There exist, for example, models where θ_{13} vanishes at the GUT scale, but RG corrections still lead to a finite value of θ_{13} at low energies. A certain finite value of θ_{13} is therefore guaranteed unless the initial values at the GUT scale and the independent RG effects are adjusted to cancel each other. From the discussion of the previous section, we would also know that the CP phase δ would vanish for all scales for Dirac neutrinos, while it could become finite for Majorana neutrinos. A finite value of δ and θ_{13} would thus exclude either Dirac neutrinos or $\theta_{13}(M_{GUT}) = 0$. Similar arguments can be made for other quantities of the leptonic mixing matrix. θ_{23}, for example, is within current experimental errors compatible with 45°. However, as in the case of θ_{13}, certain deviations are expected from RG effects even if 45° is exactly predicted at the GUT scale. Future precision measurements of neutrino oscillations may therefore allow interesting tests of flavor models and related renormalization group effects.

Atmospheric neutrino data [34] and results from the K2K long-baseline accelerator experiment [34] currently determine $\Delta m^2_{31} = (2.2^{+0.6}_{-0.4}) \times 10^{-3}\,\mathrm{eV}^2$ and $\theta_{23} \approx 45°$ [34, 35], whereas solar neutrino data [36–43], combined with the results from the KamLAND reactor experiment [44] lead to $\Delta m^2_{21} = (8.2^{+0.3}_{-0.3}) \times 10^{-5}\,\mathrm{eV}^2$ and $\tan^2\theta_{12} = 0.39^{+0.05}_{-0.04}$ [35]. These results are to a good approximation still described by two independent two flavor oscillations. The key parameter for genuine three flavor effects is the mixing angle θ_{13} which is so far only known to be small from the CHOOZ [45, 46] and Palo Verde [47]

experiments. The current bound for θ_{13} depends on the value of the atmospheric mass squared difference and it gets weaker for $\Delta m^2_{31} \lesssim 2 \times 10^{-3}\,\mathrm{eV}^2$. However, in that region an additional constraint on θ_{13} from global solar neutrino data becomes important [48,49]. At the current best fit value of $\Delta m^2_{31} = 2.2 \times 10^{-3}\,\mathrm{eV}^2$ the 3σ bound is $\sin^2\theta_{13} \leq 0.041$ [35].

Genuine three flavor oscillation effects occur only for a finite value of θ_{13} and establishing a finite value of θ_{13} is therefore one of the next milestones in neutrino physics. Leptonic CP violation is another three flavor effect which can only be tested if θ_{13} is finite. There exists therefore a very strong motivation to establish a finite value of θ_{13} and then leptonic CP violation [50–54]. Different experimental projects are therefore under construction or are being planned in order to achieve these goals. It is useful to divide the future into what can be achieved with specific current or next generation projects and what may be achieved with long term projects. The MINOS [55] project, which started already data taking, and the CNGS projects ICARUS [56] and OPERA [57], which are completing construction can be considered as "current projects". Beyond that exist other, more ambitious "next generation" projects like the superbeam experiments J-PARC to SuperKamiokande (T2K) [58] and the NuMI off-axis experiment NOνA [59]. In addition there are "next generation" plans for new reactor neutrino experiments [60] with a near and far detector. A first interesting question concerns improvements of Δm^2_{31} and $\sin^2\theta_{23}$. In Tab. 1 we show the relative precision which can be obtained in the future in comparison to the current precision, as obtained from a global fit to SuperKamiokande (SK) atmospheric and K2K long-baseline data [48, 49]. We observe from these numbers, that the accuracy on Δm^2_{31} can be improved by one order of magnitude, whereas the accuracy on $\sin^2\theta_{23}$ will be improved only by a factor two.

	$\lvert\Delta m^2_{31}\rvert$	$\sin^2\theta_{23}$
current	88%	79%
MINOS+CNGS	26%	78%
T2K	12%	46%
NOνA	25%	86%
Combined	9%	42%

Table 1: Relative precision of $|\Delta m^2_{31}|$ and $\sin^2\theta_{23}$ at 3σ for the values $\Delta m^2_{31} = 2 \times 10^{-3}\,\mathrm{eV}^2$, $\sin^2\theta_{23} = 0.5$. The last row is the relative precision which can be obtained by combining all experiments (from [61]).

Tab. 1 depends on the value of Δm^2_{31} and the sensitivity suffers for all experiments for low values of Δm^2_{31}. T2K will provide a relatively precise determination of Δm^2_{31} for $\Delta m^2_{31} \gtrsim 2 \times 10^{-3}\,\mathrm{eV}^2$. Although NO$\nu$A can put a comparable lower bound on Δm^2_{31}, the upper bound is significantly weaker, and similar to the bound from MINOS [61]. The reason for this is a strong correlation between Δm^2_{31} and θ_{23}, which disappears only for $\Delta m^2_{31} \gtrsim 3 \times 10^{-3}\,\mathrm{eV}^2$. From Tab. 1 one can also see that only T2K is able to improve the current bound on $\sin^2\theta_{23}$. The main reason for the rather poor performance on $\sin^2\theta_{23}$ is the fact that these experiments are mostly sensitive to $\sin^2 2\theta_{23}$. This implies that for $\theta_{23} \approx \pi/4$ it is very hard to achieve a good accuracy on $\sin^2\theta_{23}$, although $\sin^2 2\theta_{23}$ can be measured with relatively high precision [62].

An important task of the next generation long baseline and reactor experiments of the coming years will be to establish the three flavored-ness of the oscillations. The sensitivity

to a finite value of the key parameter θ_{13} is only modest for MINOS, OPERA and ICARUS. Double Chooz, T2K and NOνA can do much better. The $\sin^2 2\theta_{13}$-limits of the beam experiments are, however, strongly affected by parameter correlations and degeneracies, whereas new reactor experiments provide a "clean" measurement of $\sin^2 2\theta_{13}$ [63]. Altogether these experiments will provide an improvement by about a factor ten for $\sin^2 2\theta_{13}$ over the existing limit. In addition, the KamLAND [44] (and solar neutrino) data will also further increase the accuracy for Δm^2_{21} and θ_{12}. An accuracy of 5% for Δm^2_{12} and 20% for $\sin^2\theta_{12}$ is expected. Further improvements are possible, e.g. by loading the SuperKamiokande detector with Gadolinium, which might lead to an error of 3% for Δm^2_{21} and 15% for $\sin^2\theta_{12}$, both at 99%CL [64].

Beyond the next generation accelerator and reactor based oscillation experiments exist much more ambitious projects like the JHF-HyperKamiokande project, beta beams and neutrino factories[5]. Such experiments clearly require further R&D before they can be built. However, assuming current knowledge, such facilities appear to be possible and they will lead to a precision at the level of percent or even below. With a neutrino factory, for example, a sensitivity to a finite value of $\sin^2 2\theta_{13}$ might be achievable down to 10^{-5}.

It is interesting to compare these perspectives with RG effects. To illustrate the RG effects, we start with initial values for the mixing parameters at the GUT scale, $M_{\mathrm{GUT}} = 3\times 10^{16}\,\mathrm{GeV}$, assuming that these values find an explanation in a more fundamental theory.[6] These initial values are then compared with the corresponding mixing parameters at m_Z. In all our illustrations, we assume $m_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$, and a normal mass hierarchy. The simple expressions (Eqs. (1)–(3) and (5)) allow a quick estimate of the RG effects. A more precise evaluation requires a numerical analysis for which we use the Mathematica package `REAP` [33], which is publicly available[7].

The mixing angles θ_{12} and θ_{23} turn out to be rather unstable for a degenerate spectrum (cf. Fig. 3). As a consequence, a Dirac version of quark-lepton complementarity [68–70] can only work for certain mass eigenvalues and ratios of the Higgs VEVs and $\tan\beta$ (for the discussion of the RG effects in the see-saw Majorana case see [33, 71, 72]). This means stability of θ_{12} is only given in models with hierarchical masses and/or small $\tan\beta$. Note also that for an inverted hierarchy θ_{12} is unstable. This means that concerning θ_{12} RG effects are in general an issue. RG corrections to the special value $\theta_{23} = 45°$ can be comparable to the precision of upcoming experiments. Again, this happens for a quite degenerate spectrum and/or large $\tan\beta$.

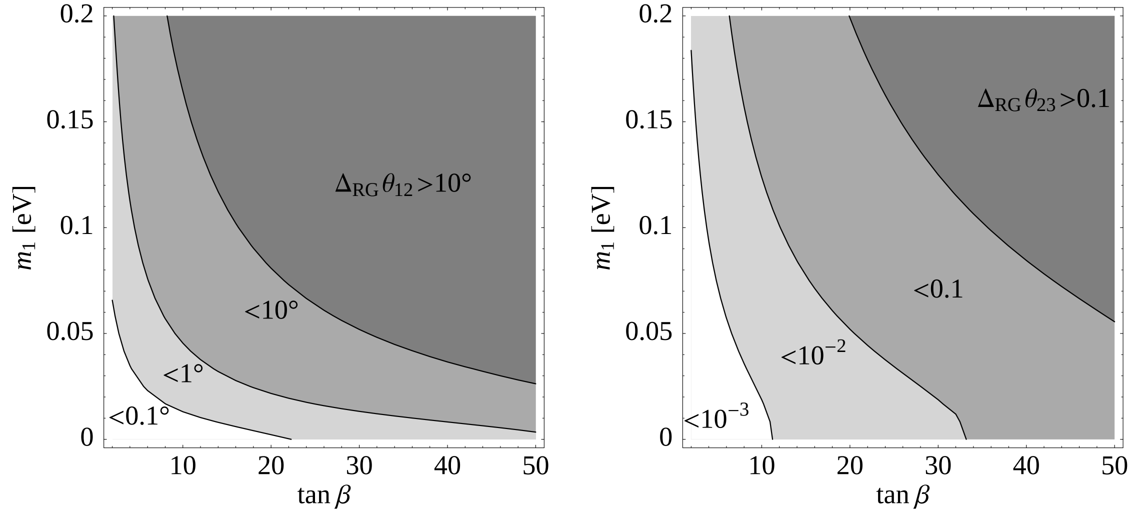
The running of θ_{13} depends crucially on its initial value. We illustrate this by plotting the radiative correction to $\sin^2 2\theta_{13}$ in Fig. 4. Most important is the second term in Eq. (2) which is dominant for not too small θ_{13}. As a consequence we find that, for $\theta_{13} = 0$ at the high scale, running in general does not generate a measurable value at the low scale. Only for the most optimistic sensitivities, a quite degenerate spectrum and large $\tan\beta$ this conclusion can be avoided. On the other hand, if θ_{13} is not tiny, RG effects can be comparable to the precision of upcoming experiments (except for small $\tan\beta$).

Finally, let us discuss corrections to δ. From the previous discussion in Sec. 2 it is clear

[5]See [65] for a comparison and for references.

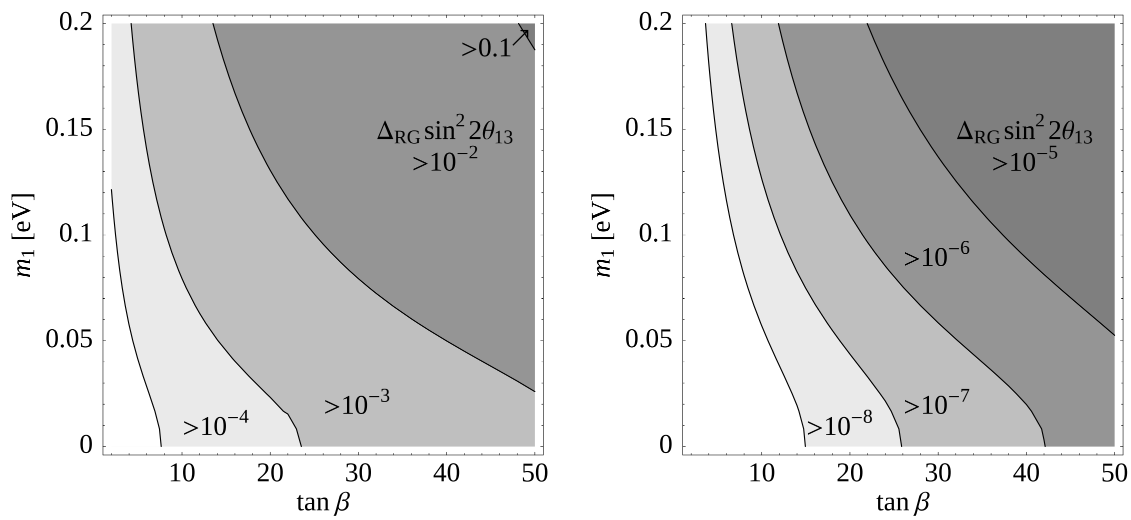
[6]One could, for instance, enjoy the possibility of fixing the initial values by continuous (e.g. [66]) or discrete (e.g. [67]) symmetries. In this case, RG effects add to the corrections arising from the breakdown of those symmetries.

[7]See `http://www.ph.tum.de/~rge/`



(a) $\Delta_{\rm RG}\theta_{12} \equiv \theta_{12}(m_Z) - \theta_{12}(M_{\rm GUT})$. (b) $\Delta_{\rm RG}\theta_{23} \equiv |0.5 - \sin^2\theta_{23}(M_{\rm GUT})|$.

Figure 3: Radiative correction to (a) θ_{12} and (b) θ_{23} for $\theta_{12} = 33°$, $\theta_{13} = 10°$, $\theta_{23} = 45°$ and $\delta = 90°$ at $\mu = m_Z$ as a function of $\tan\beta$ and m_1. These contours remain to a large extent unchanged when varying θ_{13} in the allowed range and δ arbitrarily.

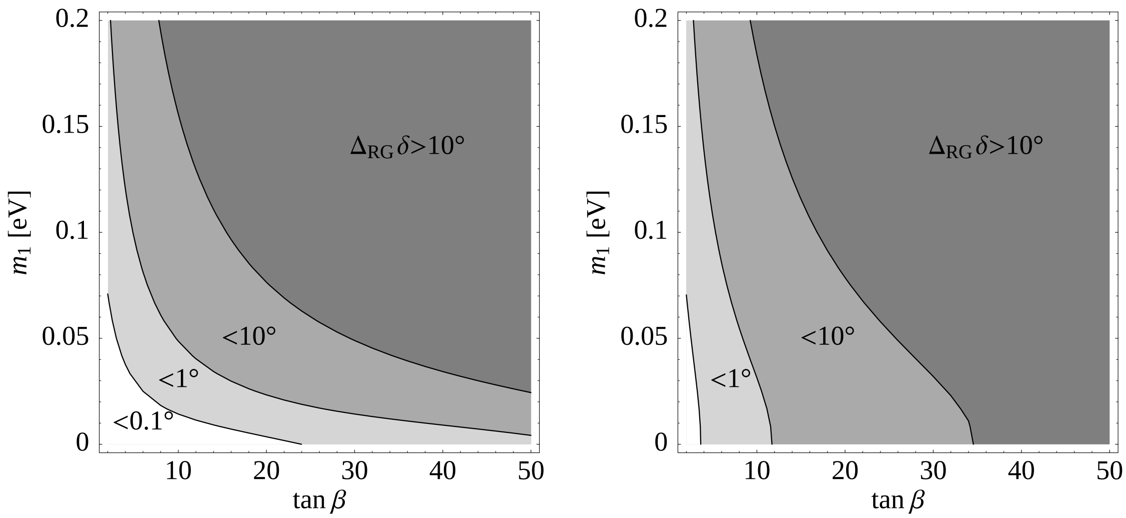


(a) $\theta_{13}(m_Z) = 10°$; $\delta(m_Z) = 90°$. (b) $\theta_{13}(m_Z) = 0$.

Figure 4: Radiative correction to $\sin^2 2\theta_{13}$, defined as $\Delta_{\rm RG}\sin^2 2\theta_{13} \equiv |\sin^2 2\theta_{13}(M_{\rm GUT}) - \sin^2 2\theta_{13}(m_Z)|$, as a function of $\tan\beta$ and m_1. We take $\theta_{12}(m_Z) = 33°$ and $\theta_{23}(m_Z) = 45°$.

that small θ_{13} corresponds to an unstable configuration with large RG effects, even for a hierarchical spectrum (cf. Fig. 5 (b)). In particular, RG effects are generically comparable with the precision of future experiments such as the combination T2K+NOνA+Reactor-II, T2HK and NuFact-II (see [73] and references therein).

Let us finally mention that RG effects for Dirac neutrinos will always result in a rescaling



(a) $\theta_{13}(m_Z) = 10°$ (b) $\theta_{13}(m_Z) = 0.1°$

Figure 5: Radiative correction to δ, defined as $\Delta_{\mathrm{RG}}\delta \equiv \left|\delta(m_Z) - \delta(M_{\mathrm{GUT}})\right|$ for (a) $\theta_{13}(m_Z) = 10°$ and (b) $\theta_{13}(m_Z) = 0.1°$ as a function of $\tan\beta$ and m_1. We use $\delta(m_Z) = 90°$.

of the mass eigenvalues. Beyond that, in the framework of the SM, mixing parameters are quite stable. The only exceptions are θ_{12} for very degenerate masses, and δ for tiny θ_{13}. On the other hand, in the MSSM, RG effects are increased by $\tan^2\beta$, i.e. by up to three orders of magnitude.

4 Summary

Assuming Dirac neutrinos, we have derived renormalization group equations for leptonic mixing parameters. The results share several features with the corresponding equations for Majorana neutrinos. However, Dirac running is more predictive, as the Majorana phases are unphysical in this case. This makes it possible to specify the amount of renormalization group evolution unambiguously as soon as m_1 and δ (and $\tan\beta$) are known. The renormalization group evolution alone does not yield an explanation of the largeness of the leptonic mixing angles (for an analogous and very clear discussion for Majorana neutrinos see [74]). Yet it may account for radiative enhancement of θ_{12}, and possibly also of θ_{23}, since both can increase significantly in the MSSM when running down.

Most importantly, we find that in phenomenological studies renormalization group effects for leptonic mixing angles can in general not be neglected. This can be understood from the fact that $\dot{\theta}_{ij} = f(m_i, \theta_{ij}, \delta)/(m_i^2 - m_j^2)$ which becomes singular if $m_i \to m_j$ and vanishes if the mixing angles are zero. We have thus traced back the relative enhancement of the quantum corrections of leptonic mixing parameters as compared to quark mixings to two reasons. First, the mass hierarchy which suppresses the renormalization group running, can be much weaker. Second, the mixing angles are larger so that the parameters are further apart from the renormalization group stable situation where all mixings are zero.

As there is no suppression of the running by phases, the renormalization group corrections should in general be taken into account even for a strong hierarchy to accommodate the precision of future experiments. Renormalization group corrections are especially relevant if the mass spectrum is non-hierarchical, and $\tan\beta$ is large in the MSSM. Hence, similarly to the case of Majorana neutrinos [27], also in the Dirac case the non-observation of deviations of the angles from special points, e.g. of θ_{12} from $\pi/4 - \vartheta_{12}$ (with ϑ_{12} being the Cabibbo angle), of θ_{13} from 0 and θ_{23} from $\pi/4$, may restrict the parameters such as the absolute neutrino mass scale. The current experimental data already has the necessary precision to indicate disfavored parameter regions. It may also point to exactly realized symmetries and our formulae can hence be used to identify possible symmetries. Whenever a symmetry is exact and fixes some mixing parameters, those mixing parameters have to be stable under quantum corrections. For instance, it has recently been pointed out [75] that for Majorana neutrinos and an inverted hierarchy the configuration $m_3 = \theta_{13} = 0$ is stable. From the analytic expressions it is obvious that this statement also applies to the Dirac case. Likewise, a quick inspection of the RGEs excludes most of the proposed symmetries from being exact. Our formulae are basis-independent, and thus allow to understand certain features of the underlying theory, such as symmetries, in a basis-independent way. We have discussed this only for the case of CP symmetry, but it is obvious how the analysis can be carried over to other symmetries. In this context, it would be interesting to see if infrared fixed points with large mixings, as discussed in [74, 76], can be obtained for (non-standard) Dirac neutrinos as well [77]. In this case, one may hope for a scenario where the large mixings are a consequence of running, and the mechanism of generation of neutrino masses is still related to the scale where gauge couplings meet.

We conclude that in the light of future precision experiments, flavor physics might enter into an era of "precision model building". It seems possible to determine the mixing parameters to a remarkable accuracy, precise enough such that flavor models and the corresponding renormalization group effects become to a certain degree distinguishable. For a specific parameter and a desired accuracy, our formulae allow to estimate the renormalization group effects, and to judge to which extent a numerical analysis is required.

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A Mixing parameters RGEs for Dirac masses

A.1 Lagrangian

The Yukawa couplings are given by

$$-\mathcal{L}_{\text{Yuk}} = (Y_\nu)_{gf}\,\overline{N_R^g}\tilde{\phi}^\dagger \ell_L^f + (Y_e)_{gf}\,\overline{e_R^g}\phi^\dagger \ell_L^f + (Y_u)_{gf}\,\overline{u_R^g}\tilde{\phi}^\dagger Q_L^f + (Y_d)_{gf}\,\overline{d_R^g}\phi^\dagger Q_L^f \tag{A.1}$$

in the SM extended by right-handed neutrinos where $\tilde{\phi} = i\sigma_2\phi^*$. In the extended MSSM, the Yukawa couplings are analogously defined in the superpotential by

$$\mathcal{W}_{\text{Yuk}} = (Y_\nu)_{gf}\,N_R^{Cg}\phi^{(2)}\epsilon^T\ell_L^f + (Y_e)_{gf}\,e_R^{Cg}\phi^{(1)}\epsilon\ell_L^f + (Y_u)_{gf}\,u_R^{Cg}\phi^{(2)}\epsilon^T Q_L^f + (Y_d)_{gf}\,d_R^{Cg}\phi^{(1)}\epsilon Q_L^f\ . \tag{A.2}$$

The left-handed lepton and quark doublets are denoted by ℓ_L and Q_L, respectively. We assume that there is no Majorana mass term for the right-handed neutrinos.

A.2 β-functions

The relevant β-functions for the down-type quark, up-type quark, charged lepton and neutrino Yukawa coupling matrices Y_d, Y_u, Y_e and Y_ν read at one-loop [78, 79]

$$(4\pi)^2\dot{Y}_d = Y_d\left\{C_d^d\,Y_d^\dagger Y_d + C_d^u\,Y_u^\dagger Y_u + \alpha_d\right\}\ , \tag{A.3a}$$

$$(4\pi)^2\dot{Y}_u = Y_u\left\{C_u^d\,Y_d^\dagger Y_d + C_u^u\,Y_u^\dagger Y_u + \alpha_u\right\}\ , \tag{A.3b}$$

$$(4\pi)^2\dot{Y}_e = Y_e\left\{C_e^e\,Y_e^\dagger Y_e + C_e^\nu\,Y_\nu^\dagger Y_\nu + \alpha_\ell\right\}\ , \tag{A.3c}$$

$$(4\pi)^2\dot{Y}_\nu = Y_\nu\left\{C_\nu^e\,Y_e^\dagger Y_e + C_\nu^\nu\,Y_\nu^\dagger Y_\nu + \alpha_\nu\right\}\ , \tag{A.3d}$$

where

$$C_d^d = \begin{cases} 3/2\ , & \text{(SM)} \\ 3\ , & \text{(MSSM)} \end{cases} \qquad C_d^u = \begin{cases} -3/2\ , & \text{(SM)} \\ 1\ , & \text{(MSSM)} \end{cases} \tag{A.4a}$$

$$C_u^d = \begin{cases} -3/2\ , & \text{(SM)} \\ 1\ , & \text{(MSSM)} \end{cases} \qquad C_u^u = \begin{cases} 3/2\ , & \text{(SM)} \\ 3\ , & \text{(MSSM)} \end{cases} \tag{A.4b}$$

$$C_e^e = \begin{cases} 3/2\ , & \text{(SM)} \\ 3\ , & \text{(MSSM)} \end{cases} \qquad C_e^\nu = \begin{cases} -3/2\ , & \text{(SM)} \\ 1\ , & \text{(MSSM)} \end{cases} \tag{A.4c}$$

$$C_\nu^e = \begin{cases} -3/2\ , & \text{(SM)} \\ 1\ , & \text{(MSSM)} \end{cases} \qquad C_\nu^\nu = \begin{cases} 3/2\ , & \text{(SM)} \\ 3\ , & \text{(MSSM)} \end{cases} \tag{A.4d}$$

and

$$\alpha_d = \begin{cases} -\frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2 + T_{\text{SM}}\ , & \text{(SM)} \\ 3\ \text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) - \frac{7}{15}\,g_1^2 - 3\,g_2^2 - \frac{16}{3}\,g_3^2\ , & \text{(MSSM)} \end{cases} \tag{A.5a}$$

$$\alpha_u = \begin{cases} -\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2 + T_{\text{SM}}\ , & \text{(SM)} \\ \text{Tr}(Y_\nu^\dagger Y_\nu) + 3\ \text{Tr}(Y_u^\dagger Y_u) - \frac{13}{15}\,g_1^2 - 3\,g_2^2 - \frac{16}{3}\,g_3^2 \quad , & \text{(MSSM)} \end{cases} \tag{A.5b}$$

$$\alpha_\ell = \begin{cases} -\frac{9}{4}g_1^2 - \frac{9}{4}g_2^2 + T_{\text{SM}}\ , & \text{(SM)} \\ 3\ \text{Tr}(Y_e^\dagger Y_e) + \text{Tr}(Y_\nu^\dagger Y_\nu) - \frac{9}{5}\,g_1^2 - 3\,g_2^2 \quad , & \text{(MSSM)} \end{cases} \tag{A.5c}$$

$$\alpha_\nu = \begin{cases} -\frac{9}{20}\,g_1^2 - \frac{9}{4}\,g_2^2 + T_{\text{SM}}\ , & \text{(SM)} \\ \text{Tr}(Y_\nu^\dagger Y_\nu) + 3\ \text{Tr}(Y_u^\dagger Y_u) - \frac{3}{5}\,g_1^2 - 3\,g_2^2\ , & \text{(MSSM)}\ . \end{cases} \tag{A.5d}$$

Here, we define $T_{\text{SM}} \equiv \text{Tr}\left[Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3\,Y_d^\dagger Y_d + 3\,Y_u^\dagger Y_u\right]$, and use GUT normalization for g_1.

A.3 General derivation

In this subsection, we will perform a general analysis applicable for any Dirac masses, and treat the evolution of lepton and quark masses and mixings only as a special case.

We derive the running of mixing parameters for a RGE of the form

$$16\pi^2\,\frac{\mathrm{d}}{\mathrm{d}t}H \;=\; F^\dagger\,H + H\,F + f\,H\ , \tag{A.6}$$

where f is real and H is Hermitean, so that we can diagonalize it in a 'reference basis',

$$U^\dagger\cdot H\cdot U \;=\; D\ . \tag{A.7}$$

In the application in the main part, F corresponds either to $C_d^u\,Y_u^\dagger Y_u + C_d^d Y_d^\dagger Y_d$ (or $C_\nu^e\,Y_e^\dagger Y_e + C_\nu^\nu Y_\nu^\dagger Y_\nu$ for the lepton sector), and H to $Y_d^\dagger Y_d$ (or $Y_\nu^\dagger Y_\nu$). The 'reference basis' is the basis where $Y_u^\dagger Y_u$ (or $Y_e^\dagger Y_e$) is diagonal at $t = t_0$. U denotes then to the CKM matrix U_{CKM} (or the MNS matrix U_{MNS}). f denotes the diagonal parts of the β-function, $f = 2\alpha_d$ (or $f = 2\alpha_\nu$).

Now we perform an analysis very similar to what is done in [27] which is based on [32,79,80]. We can differentiate the relation $H = U\cdot D\cdot U^\dagger$,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(U\cdot D\cdot U^\dagger) \;&=\; \dot{U}\cdot D\cdot U^\dagger + U\cdot D\cdot\dot{U}^\dagger + U\cdot\dot{D}\cdot U^\dagger \\ &\overset{!}{=}\; \frac{1}{16\pi^2}\left(F^\dagger\cdot U\cdot D\cdot U^\dagger + U\cdot D\cdot U^\dagger\cdot F + f\,U\cdot D\cdot U^\dagger\right)\ .\end{aligned} \tag{A.8}$$

Multiplying by $U^\dagger$ from the left and by U from the right yields

$$U^\dagger\cdot\dot{U}\cdot D + D\cdot\dot{U}^\dagger\cdot U + \dot{D} \;=\; \frac{1}{16\pi^2}\left(F'^\dagger\cdot D + D\cdot F' + f\,D\right)\ , \tag{A.9}$$

where $F' = U^\dagger\cdot F\cdot U$. For the quark case, $F' = C_d^d\,D + C_d^u\,U^\dagger Y_u^\dagger Y_u\,U$. We will see below that only the off-diagonal components are relevant for the RGEs of the mixing parameters.

The evolution of U can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}U \;=\; U \cdot X \;, \tag{A.10}$$

where X is anti-Hermitean. Inserting this relation yields

$$\dot{D} + X \cdot D + D \cdot X^\dagger \;=\; \frac{1}{16\pi^2}\left(F'^\dagger \cdot D + D \cdot F' + f\, D\right) \;, \tag{A.11}$$

or, by using the anti-Hermitecity of X,

$$\dot{D} \;=\; \frac{1}{16\pi^2}\left(f\, D + F'^\dagger \cdot D + D \cdot F'\right) - X \cdot D + D \cdot X \;. \tag{A.12}$$

Denoting the entries of D by y_i^2, i.e. $D = \mathrm{diag}(y_1^2, y_2^2, y_3^2)$, we find

$$\frac{\mathrm{d}}{\mathrm{d}t}y_i^2 \;=\; \frac{1}{16\pi^2}\left[f\, y_i^2 + (F'^*_{ii} + F'_{ii})\, y_i^2\right] \;, \tag{A.13}$$

i.e. the terms proportional to X have dropped out. This equation corresponds to a RGE for the running mass eigenvalues, defined by $m_i(t) = |y_i(t)|\, v$ with v fixed, of the form

$$(4\pi)^2 \dot{m}_i \;=\; (\mathrm{Re}\, F'_{ii} + \alpha)\, m_i \;. \tag{A.14}$$

By analyzing the off-diagonal parts we obtain

$$y_i^2\, X_{ij} - X_{ij}\, y_j^2 \;=\; -\frac{1}{16\pi^2}\left[(F'^\dagger)_{ij}\, y_j^2 + y_i^2\, F'_{ij}\right] \;. \tag{A.15}$$

This can be converted into equations for real and imaginary part of X, which, since F is Hermitean, can be combined to

$$16\pi^2 X_{ij} = \frac{y_j^2 + y_i^2}{y_j^2 - y_i^2}\, F'_{ij} \;. \tag{A.16}$$

The diagonal parts of X remain undetermined. However, this is not a problem, since they only influence the RG evolution of the unphysical phases.[8]

So far, we have derived the differential change of the CKM matrix due to the RG corrections for $Y_d^\dagger Y_d$ (cf. Eq. (A.10)). In the Majorana neutrino case, the analogous differential equation already describes the evolution of the MNS matrix since $Y_e^\dagger Y_e$ doesn't get rotated by the RGE.[9] For Dirac neutrinos, $Y_e^\dagger Y_e$ gets rotated only by terms proportional to the squares of Dirac Yukawa couplings, hence those rotations can safely be neglected. In the quark sector, the radiative rotation of $Y_u^\dagger Y_u$ represents an important effect, as we will argue in the following.

[8]Note that the Majorana phases are unphysical in the the Dirac case as well.

[9]This is only true at leading order in M^{-1} where M denotes the scale of the effective neutrino mass operator (e.g. the see-saw scale) [81].

A.4 Contribution from the change of Y_u

Here, we specialize to the quark sector as the analogous effect is irrelevant for Dirac neutrinos.

The RGE for Y_u contains non-diagonal terms so that continuous re-diagonalization is required. Since the mixing matrix U_{CKM} is defined as the matrix which diagonalizes $Y_d^\dagger Y_d$ in the basis in which Y_u is diagonal, U_{CKM} receives an additional contribution from the running of Y_u,

$$\frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{CKM}} = U_{\mathrm{CKM}} \cdot X + \text{term stemming from the change of } Y_u \ . \tag{A.17}$$

To evaluate this change, we can essentially repeat the steps of the previous subsection. Introducing a matrix $\widetilde{U}$ which diagonalizes $Y_u^\dagger Y_u$ in the reference basis (implying $\widetilde{U}(t = t_0) = \mathbb{1}$), i.e.

$$\widetilde{U}^\dagger \, Y_u^\dagger Y_u \, \widetilde{U} = \mathrm{diag}(\widetilde{y}_1^2, \widetilde{y}_2^2, \widetilde{y}_3^2) \ , \tag{A.18}$$

we arrive at

$$\frac{\mathrm{d}}{\mathrm{d}t} \widetilde{U} = \widetilde{U} \cdot \widetilde{X} \ , \tag{A.19}$$

where the off-diagonal entries of $\widetilde{X}$ are given by

$$16\pi^2 \, \widetilde{X}_{ij} = \frac{\widetilde{y}_i^2 + \widetilde{y}_j^2}{\widetilde{y}_j^2 - \widetilde{y}_i^2} \, \widetilde{F}_{ij} \ . \tag{A.20}$$

Completely analogous to A.3,

$$\widetilde{F}' = \widetilde{U}^\dagger \cdot \widetilde{F} \cdot \widetilde{U} \ , \tag{A.21}$$

and at $t = t_0$

$$\widetilde{F}' = C_u^d \, U \, D \, U^\dagger + C_u^u \, Y_u^\dagger Y_u \ . \tag{A.22}$$

Again, only the off-diagonal terms influence the RGEs of the mixing angles.

A.5 Mixing parameter RGEs in the quark sector

As $U_{\mathrm{CKM}} = \widetilde{U}^{-1} U = \widetilde{U}^\dagger U$, the RGE for the CKM matrix reads

$$\frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{CKM}} = \widetilde{X}^\dagger \cdot U_{\mathrm{CKM}} + U_{\mathrm{CKM}} \cdot X \ . \tag{A.23}$$

To proceed, we label the mixing parameters by

$$\{\xi_k\} = \{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \delta_e, \delta_\mu, \delta_\tau, \varphi_1, \varphi_2\} \ , \tag{A.24}$$

and evaluate the derivative of U_{CKM},

$$\dot{U}_{\mathrm{CKM}} = \dot{U}_{\mathrm{CKM}} \left(\{\dot{\xi}_k\}, \{\xi_k\} \right) \ . \tag{A.25}$$

Observe that the resulting expression is linear in $\dot{\xi}_k$. By solving a system of linear equations of the form

$$\sum_k A_{TX}^{(k)}\,\dot{\xi}_k + \mathrm{i}\,S_{TX}^{(k)}\,\dot{\xi}_k \;=\; R_X\;, \tag{A.26}$$

where

$$R_X \;=\; U_\mathrm{MNS}\cdot T + X^\dagger\cdot U_\mathrm{MNS}\;, \tag{A.27}$$

we thus obtain a set of linear equations for the $\dot{\xi}_k$. RGEs for the matrix elements have been derived in refs. [79, 80].

From these, we obtain the RGEs for the mixing angles in the quark sector. Neglecting all Yukawa coupling except for y_t and y_b, they read

$$\begin{aligned}
\dot{\vartheta}_{12} \;&=\; \frac{C_d^u\,{y_t}^2}{64\,\pi^2}\,\cos(\vartheta_{12})\,\left\{\left[(3-\cos 2\,\vartheta_{13})\,\cos 2\,\vartheta_{23} - 2\,\cos^2\vartheta_{13}\right]\,\sin\vartheta_{12}\right.\\
&\qquad\left. +\,4\,\cos\delta_\mathrm{CP}\,\cos\vartheta_{12}\,\sin\vartheta_{13}\,\sin 2\,\vartheta_{23}\right\}\;, &\text{(A.28a)}\\
\dot{\vartheta}_{13} \;&=\; \frac{-\sin 2\,\vartheta_{13}}{64\,\pi^2}\,\left[2\,C_u^d\,y_b^2 + C_d^u\,y_t^2\,\left(1+\cos 2\,\vartheta_{23}\right)\right]\;, &\text{(A.28b)}\\
\dot{\vartheta}_{23} \;&=\; \frac{-\sin 2\,\vartheta_{23}}{64\,\pi^2}\,\left[C_u^d\,y_b^2\,\left(1+\cos 2\,\vartheta_{13}\right) + 2\,C_d^u\,y_t^2\right]\;. &\text{(A.28c)}
\end{aligned}$$

It turns out that finite y_s and y_c corrections yield an important but sub-dominant effect for $\dot{\vartheta}_{12}$. The dominant term in the RGE of δ_CP is

$$\dot{\delta}_\mathrm{CP} \;=\; \frac{C_d^u\,y_s^2\,y_t^2}{8\,\pi^2\,\left(y_b^2-y_s^2\right)}\,\cos\vartheta_{12}\,\cos\vartheta_{23}\,\sin\delta\,\sin\vartheta_{12}\,\sin\vartheta_{23}\times\vartheta_{13}^{-1}\;. \tag{A.29}$$

A.6 Mixing parameter RGEs in the (Dirac) neutrino sector

In order to derive analogous RGEs for the leptonic mixing parameters, observe that the RG change of $Y_e^\dagger Y_e$ is quadratic in neutrino Yukawa couplings, i.e. strongly suppressed. Thus, we can safely neglect the $\widetilde{X}$ contribution,

$$\frac{\mathrm{d}}{\mathrm{d}t}U_\mathrm{MNS} \;=\; \widetilde{X}^\dagger\cdot U_\mathrm{MNS} + U_\mathrm{MNS}\cdot X \;\simeq\; U_\mathrm{MNS}\cdot X\;, \tag{A.30}$$

where X is now related to F' by Eq. (A.16), and $F' = C_\nu^\nu\,D + C_\nu^e\,U_\mathrm{MNS}^\dagger Y_e^\dagger Y_e\,U_\mathrm{MNS}$ at $t=t_0$.

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